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A Table of Class Numbers for Cubic Number Fields.

By Legh W. Reid.

This table has been calculated with a view to furnishing for the general algebraic number fields an amount of number material sufficiently great to be of use in the further study of these fields, and in particular in that of the cubic fields. It gives for each of 161 cubic number fields the class number, h, the discriminant Δ, a basis, and the factorization of certain rational primes into their prime ideal factors. When $h = 1$, the prime *number* factors of these primes are given. Units are also given for most of the fields. The method employed in the calculation of the class numbers is to be looked at from the point of view of the practicability of carrying out the numerical reckoning involved, and the actual determination of the numerical value of h. It is to be sharply distinguished from those representations of h by an infinite series, which, although theoretically perfect, lead in only a very limited number of cases to the determination of the numerical value of h.

The method used depends upon the following theorem of Minkowski's:

Th. 1.—*In every ideal class there is an ideal,* $\mathfrak{i}$, *whose norm,* $n(\mathfrak{i})$, *satisfies the condition*

$$n(\mathfrak{i}) < \left(\frac{4}{\pi}\right)^r \frac{m!}{m^m} \left| \sqrt{\Delta} \right|, \tag{1}$$

where m is the degree and Δ the discriminant of the field, and r the number of pairs of imaginary fields found among the m conjugate fields $k^{(1)}, k^{(2)}, \ldots, k^{(m)}$. I shall denote $\left(\frac{4}{\pi}\right)^r \frac{m!}{m^m}$ by M. If then we find all ideals of a proposed field, k, whose norms satisfy the condition (1), and determine the equivalences which exist between them; i. e., into how many ideal classes they fall, we have determined the class number of k.

The task may be divided into two parts:

I. The obtaining of all ideals whose norms satisfy (1).

II. The determination of the number of the ideal classes into which they fall.

Let θ be an integer defining the body k, $f(x)=0$, the equation of lowest degree, the m^{th}, with rational coefficients, satisfied by θ, and $d(\theta)$ the discriminant of this equation.

We must first of all determine a basis and the discriminant Δ of k.

It can be easily shown that if $d(\theta)$ be not divisible by the square of a rational integer, then

$$d(\theta)=\Delta$$

and $1, \theta^2, \ldots, \theta^{m-1}$ is a basis of k.

In the case of *cubic bodies*, when $d(\theta)$ is divisible by the square of a rational integer, we may determine a basis and hence Δ by a method given by Woronoj.

I. We obtain all ideals of k, whose norms satisfy (1) in the following manner: Since the norm of a prime ideal is a power of the rational prime, which it divides, we shall obtain all prime ideals, whose norms satisfy (1), if we factor into their prime ideal factors all rational primes $< M|\sqrt{\Delta}|$.

The desired ideals are then such of these prime ideals, their powers and products as satisfy (1). We have, then, first of all to factor all rational primes $< M|\sqrt{\Delta}|$.

This is easily accomplished in the case $\dfrac{d(\theta)}{\Delta} \not\equiv 0 \pmod p$ by means of the following theorem:

TH. 2.—*If p satisfy the condition $\dfrac{d(\theta)}{\Delta} \not\equiv 0 \pmod p$, and if we resolve the left-hand member of $f(x)=0$ into its prime factors with respect to the modulus p, as*

$$f(x) \equiv \{P(x)\}^e \{P'(x)\}^{e'}, \ldots, \pmod p,$$

where $P(x)$, $P'(x)$, ... are different prime functions with respect to p, and of degrees f, f', ... respectively, then is

$$(p)=(p,\, P(\theta))^e\,(p,\, P'(\theta))^e \ldots$$

the required factorization of (p), where $(p, P(\theta))$, $(p, P'(\theta))$ are different prime ideals of degrees f, f', ... respectively.

When $\frac{d(\theta)}{\Delta} \equiv 0 \pmod{p}$, the factorization of (p) may be effected in the case of *cubic bodies* by a method given by Woronoj.

I shall consider, during the remainder of this paper, the body under discussion to be cubic.

II. We now take up the determination of the number of ideal classes into which fall the ideals of k whose norms satisfy (1).

The method that I have used is the following:

Having selected any prime ideal $\mathfrak{p}$ whose norm satisfies the above condition, we must determine first of all whether $\mathfrak{p}$ is a principal ideal. The following method answers not only this question but the more general one. What is the lowest power of $\mathfrak{p}$, which is a principal ideal?

We find an integer α such that $(\alpha) = \mathfrak{p}^n$. $\mathfrak{p}$ is evidently a principal or non-principal ideal according as (α) is or is not the n^{th} power of a principal ideal. If $n = 1$, $\mathfrak{p} = (\alpha)$, a principal ideal.

The necessary and sufficient condition that (α) be the n^{th} power of a principal ideal is that a unit η exist such that $\alpha\eta$ is the n^{th} power of an integer.

That is (α) can be the n^{th} power of a principal ideal, although α is not the n^{th} power of an integer, but acquires this property through multiplication by a suitable unit.

The following theorems simplify the determination of the lowest power of $\mathfrak{p}$, which is a principal ideal:

TH. 3.—*If the m_1^{th} and m_2^{th} powers of an ideal $\mathfrak{a}$ be principal ideals, then is also the l^{th} power of $\mathfrak{a}$ a principal ideal, where l is the greatest common divisor of m_1 and m_2.*

TH. 4.—*The necessary and sufficient condition, that $\mathfrak{a}^n = (\alpha)$ be the lowest power of $\mathfrak{a}$, which is a principal ideal, is that (α) be neither the p_1^{th}, p_2^{th}, $\ldots$, nor p_r^{th} power of a principal ideal, where $p_1, p_2, \ldots, p_r$ are the different prime factors of n.*

The problem of determining the lowest power of $\mathfrak{p}$, which is a principal ideal, be reduced, therefore, to that of determining whether a given principal ideal be the p^{th} power of a principal ideal, where p is a prime number.

To determine, therefore, whether (α) be the p^{th} power of a principal ideal, we must multiply α with each one of a system of units,

$$\eta_1, \eta_2, \ldots, \eta_r$$

such that if α be not the p^{th} power of a principal ideal but acquire this property through multiplication by a unit η, then one of the units of this system will be η, and hence one of these products will be the p^{th} power of an integer.

Such a system of units I call a *complete unit system* for the power p. (α), therefore, is or is not the p^{th} power of a principal ideal according as *one* or *none* of the products

$$\alpha, \; \alpha\eta_1, \; \alpha\eta_2, \; \ldots, \; \alpha\eta_r$$

is the p^{th} power of a principal ideal, where $\eta_1, \ldots, \eta_r$ is a complete unit system for the power p. In the construction of such a system of units in the case of a cubic field, we must distinguish two cases, according as Δ is negative or positive, that is, according as among the conjugate fields k, k', k'' there are two or no imaginary fields, and k has respectively one or two fundamental units. Confining ourselves to the case Δ negative and p an odd prime, such a system of units may be constructed for the power p as follows. All units of the field have the form $\eta = \pm \varepsilon^m$, where ε is a fundamental unit.

If α be the p^{th} power of an integer or acquire this property through multiplication by a unit, it has the form, when $p \neq 2$,

$$\alpha = \beta^p \varepsilon^l.$$

If we take $\eta = \pm \varepsilon^g$, where $g \equiv -l \pmod{p}$, then

$$\alpha\eta = \pm \beta^p \varepsilon^{l+g}.$$

The p units

$$\eta = \varepsilon^g \; (\text{or} - \varepsilon^g), \quad g \equiv 0, \; 1, \; 2, \; \ldots, \; p-1, \pmod{p}$$

form a complete unit system for the power p. To obtain such a system we have only to find a unit η, which is not the p^{th} power of a unit, i. e.,

$$\eta = \varepsilon^g \; (\text{or} - \varepsilon^g), \; g \not\equiv 0, \pmod{p}$$

It is evidently indifferent whether we have $\eta = \varepsilon^g$ or $\eta = -\varepsilon^g$ since p is odd. For the sake of simplicity, we shall take the $+$ sign, though the unit found might have the $-$ sign. $\eta^0 = 1, \; \eta, \; \eta^2, \; \ldots, \; \eta^{p-1}$ constitute then a complete unit system for the power p, since $0, g, 2g, \ldots, (p-1)\,g$ form a complete remainder system with respect to the modulus p, and hence one of these exponents is $\equiv -l \pmod{p}$. (α), therefore, is or is not the p^{th} power of a principal ideal according as one or none of the products

$$\alpha, \; \alpha\eta, \; \alpha\eta^2, \; \ldots, \; \alpha\eta^{p-1}$$

is the p^{th} power of an integer. When $p = 2$, we have merely in addition to the above to take into consideration the multiplication of α by the unit -1, which is unnecessary when p is odd, since then $-\alpha$ is or is not the p^{th} power of an integer according as α is or is not the p^{th} power of an integer. When Δ is positive, the method is similar, account simply being taken of the fact that the field has two fundamental units.

In order to determine whether any one of the above products be the p^{th} power of an integer, we set it equal to $(a\omega_1 + b\omega_2 + c)^p$, where ω_1, ω_2, 1 is a basis of k. We then obtain, by equating the coefficients of the corresponding powers of θ on the two sides of the equation, three equations to determine a, b and c. The necessary and sufficient condition for the product under discussion to be the p^{th} power of an integer is that these equations have an integral solution.

By means now of Theorem 4 and the above method for determining whether a given principal ideal is the p^{th} power of a principal ideal, we can determine the lowest power of $\mathfrak{p}$, which is a principal ideal. Let this power be the t^{th}, then

$$\mathfrak{p},\ \mathfrak{p}^2,\ \ldots\ ,\ \mathfrak{p}^{t-1},\ \mathfrak{p}^t \sim (1)$$

are representatives of t different ideal classes, which we denote with

$$A,\ A^2,\ \ldots\ ,\ A^{t-1},\ A^t \sim 1.$$

The class number, h, must now be divisible by t. Let N be the number of ideals whose norms satisfy (1), the unit ideal (1) being included.

If $N < 2t$, we have at once $h = t$.

If, however, $N \nless 2t$, we determine the classes of some of the remaining ideals satisfying 1.

Let $\mathfrak{j}$ be one of them. If we can find a principal ideal (γ) such that $(\gamma) = \mathfrak{p}^r\mathfrak{j}$, then $\mathfrak{j}$ belongs to the class which is reciprocal to that of $\mathfrak{p}^r$, i. e., to $A^{r'}$, where $r' \equiv -r \pmod{t}$. We can then easily determine in which classes lie the different powers of $\mathfrak{j}$, and the products of these powers, with those of $\mathfrak{p}$. If we cannot find such a principal ideal (γ), we must determine the lowest power of $\mathfrak{j}$. which is a principal ideal, exactly as in the case of $\mathfrak{p}$. Let this power be the s^{th}. We must now determine whether $\mathfrak{j}$ lies in any one of the classes A, A^2, ... , A^{t-1}, It can be shown that it is possible for $\mathfrak{j}$ to lie in one of these classes only when we have $t \equiv 0 \pmod{s}$, in which case, if $\frac{t}{s} = t'$, it is possible to have $\mathfrak{j} \sim \mathfrak{p}^{i't'}$, where i' is prime to s. There are, therefore, at most $\phi(s)$ classes, in one of

which $\mathfrak{j}$ may lie. If $s \equiv 0 \pmod{t}$, it would be possible likewise for $\mathfrak{p}$ to lie in one of $\phi(t)$ of the classes $B, B^2, \ldots, B^{s-1}$ represented by the powers of $\mathfrak{j}$. To determine whether $\mathfrak{j}$ lies in the class A^i, we must determine whether the product of $\mathfrak{j}$ and an ideal $\mathfrak{h}$ belonging to the class, which is reciprocal to that of A^i, is a principal ideal. To do this we must find a principal ideal (λ), which is a power of $\mathfrak{j}\mathfrak{h}$, and proceed as already indicated. If $\mathfrak{j}$ belongs to none of the classes $1, A, \ldots, A^{t-1}$, we have st different classes. If $N < 2st$, then $h = st$, and in general, letting, at any point of the reckoning, n be the number of the ideals satisfying 1 whose classes have been determined, k the number of the known classes which have found representatives among these ideals, and K the number of the known classes, from

$$N - n + k < 2K \tag{2}$$

follows

$$h = K.$$

The use of (2) saves much reckoning, as we find from it that we have to determine the classes of only $N' + 1$ of the ideals satisfying (1), where $N = 2N'$, or $2N' + 1$.

The table is arranged as follows: Part I contains all fields defined by the root of an equation of the form

$$x^3 + A_2 x + A_3 = 0,$$

where A_2, A_3 are rational integers less in absolute value than 10. The first column contains a number for purposes of reference; the second, the equation whose root defines the field; the third, the discriminant of the equation; the fourth and fifth, the discriminant of the field; the sixth, the class-number; the seventh, a basis (when no numbers stand here, $\theta^2, \theta, 1$ are to be understood); the eighth, one or more units. When, after a unit η, $\neq k^2$ is placed, this denotes that neither η nor $-\eta$ is the square of a unit. Then follow the factorizations of those rational primes $p < M|\sqrt{\Delta}|$. The factors are arranged according to the magnitude of their norms; i. e., the factor with greatest norm stands second. When $h = 1$, the factors are chosen so that their norms are positive and their product is equal to p without multiplication by a unit, except when p is divisible by the cube of a prime number. In this case, the unit is given with which the cube of the prime must be multiplied to obtain p. These units are designated by * attached to the parentheses in which they stand.

$p = p$ or $(p) = (p)$ means that p is unfactorable. When $\equiv$ and a number stand in the seventh column, it signifies that this field is identical with the field designated by the number.

Part II contains all fields defined by a root of a cubic equation of the form

$$A_0 x^3 + A_1 x^2 + A_2 x + A_3 = 0,$$

where A_0, A_1, A_2, A_3 are rational integers less in absolute value than 3, with the exception of those equations of this form which are found in Part I or are transformable into one of those of Part I by the substitution $x : x - \frac{A_1}{3A_0}$. In the cases also where one of these equations is transformable into another of the same form by a linear substitution, one only of the two is given.

Part II is arranged like Part I, with the exception that in 12, 16, 17, 18, 19, since the roots of these equations are not integers, the equations written immediately under have been used to define these fields. In each case, this equation is obtained from the original one by the substitution $x : \frac{x}{2}$.

A fuller discussion of the methods employed in the calculation of this table, with numerical examples, will be found in "Tafel der Klassenanzahlen für kubische Zahlenkörper," published by the author of this article as dissertation.

See Hilbert, "Bericht über die Theorie der algebraischen Zahlkörper," §§ 11, 24. Jahresbericht der deutschen Mathematiker-Vereinigung, Vierter Band, 1894-95.

Minkowski, "Geometrie der Zahlen."

Woronoj, "The algebraic integers, which are functions of a root of an equation of the 3d degree." (Translation of the Russian title.)

PRINCETON, N. J.

PART I.

		$d(\theta)$	Δ		h	Basis.	Units.	Factorization of Rational Primes.
1	$x^3+1=0$	red.						1 :
2	$x^3+x+1=0$	-31	-31	-31	1	......	$\theta,\ \theta+1$	$2:2\ =2\ \ 3=(-\theta+1)(\theta^2+\theta+2)\ \ 5=5$
3	$x^3-x+1=0$	-23	-23	-23	1	......	$\theta,\ \theta-1,\ \theta+1$	$3:2\ =2\ \ 3=3$
4	$x^3+2=0$	$-2^2.3^3$	$-2^2.3^3$	-108	1	. ...	$\theta+1,\ \theta^2-\theta+1$	$4:2\ =(-\theta)^3\ 3=(-\theta+1)^3(-\theta-1)^*\ \ 5=(\theta^2+1)(-\theta^2-2\theta+1)\ \ 7=7$
5	$x^3+x+2=0$	red.						5 :
6	$x^3-x+2=0$	$-2^3.13$	$-2^3.13$	-104	1	......	$\theta^2+\theta-1$	$6:2\ =(-\theta+1)^2(\theta^2-2)\ \ 3=3\ \ 5=5\ \ 7=7$
7	$x^3+2x+2=0$	$-2^2.5.7$	$-2^2.5.7$	-140	1	......	$\theta+1$	$7\begin{cases}2\ =(-\theta)^3(\theta^2-\theta+3)^*\ \ 3=3\ \ 5=(-\theta+1)^2(\theta^2+1)\\ 7\ =(2\theta^2-1)^2(52\theta^2-40\theta+135)\end{cases}$
8	$x^3-2x+2=0$	$-2^2.19$	$-2^2.19$	-76	1	......	$\theta-1,\ \theta^2+\theta-1$	$8:2\ =(-\theta)^3(\theta^2+\theta-1)^*\ \ 3=(-\theta-1)(\theta^2-\theta-1)\ \ 5=5\ \ 7=7$
9	$x^3+2x+1=0$	-59	-59	-59	1	......	θ	$9:2\ =(\theta+1)(\theta^2-\theta+3)\ \ 3=3\ \ 5=5\ \ 7=7$
10	$x^3-2x+1=0$	red.						10 :
11	$x^3+3=0$	-3^5	-3^5	-243	1	......	θ^2-2	$11\begin{cases}2\ =(-\theta-1)(\theta^2-\theta+1)\ \ 3=(-\theta)^3\ \ 5=(\theta+2)(\theta^2-2\theta+4)\ \ 7=7\\ 11=(-\theta+2)(\theta^2+2\theta+4)\ \ 13=13\end{cases}$
12	$x^3+x+3=0$	-13.19	$-13.\ 19$	-247	1	......	$\theta+1$	$12:2\ =2\ \ 3=(-\theta)(\theta^2+1)\ \ 5=(-\theta+1)(\theta^2+\theta+2)\ \ 7=(\theta+2)(\theta^2-2\theta+5)$
13	$x^3-x+3=0$	-239	-239	-239	1	.....	$\theta^2+\theta-1$	$13:2\ =2\ \ 3=(-\theta)(-\theta+1)(-\theta-1)\ \ 5=5$
14	$x^3+2x+3=0$	red.						14 :
15	$x^3-2x+3=0$	-211	-211	-211	1	......	$\theta^2-2\theta+2$	$15:2\ =(-\theta+1)(\theta^2+\theta-1)\ \ 3=(-\theta)(\theta^2-2)$
16	$x^3+3x+3=0$	$-3^3.13$	$-3^3.13$	-351	1	......	$\theta+1$	$16\begin{cases}2\ =2\ \ 3=(-\theta)^3(\theta^2-\theta+4)^*\ \ 5=5\ \ 7=(-\theta+1)(\theta^2+\theta+4)\\ 11=(\theta+2)(2\theta+1)(-3\theta^2+2\theta-11)\ \ 13=(2\theta^2-\theta+7)^2(2\theta^2-\theta-2)\\ 17=(-\theta+2)(\theta^2+2\theta+7)\end{cases}$
17	$x^3-3x+3=0$	$-3^3.5$	$-3^3.5$	-135	1	......	$\theta-1,\ \theta+2$	$17:2\ =2\ \ 3=(-\theta)^3(\theta^2+\theta-2)^*\ \ 5=(-\theta-1)^2(-\theta+2)$
18	$x^3+3x+1=0$	$-3^3.5$	$-3^3.5$	-135	1	......	$\theta,\ 3\theta+1,\ \theta^2+3$	$18:2\ =2\ \ 3=(-\theta-1)^3(3\theta^2-\theta+9)^*\ \ 5=(2\theta+1)^2(12\theta^2-4\theta+37)$
19	$x^3-3x+1=0$	3^4	3^4	81	1	......	$\theta,\ \theta-1,\ \theta+2$	$19:2\ =2\ \ 3=(-\theta-1)^3(\theta^2-\theta-1)^*\ \ 5=5\ \ 7=7$
20	$x^3+3x+2=0$	$-2^3.3^3$	$-2^3.3^3$	-216	1	......	$-\theta^2+\theta+1$	$20\begin{cases}2\ =(\theta+1)^2(3\theta^2-2\theta+10)\ \ 3=(-2\theta-1)^3(104\theta^2-62\theta+349)^*\ \ 5=5\\ 7\ =7\ \ 11=11\end{cases}$
21	$x^3-3x+2=0$	red.						21 :
22	$x^3+4=0$	$-2^4.3^3$	$-2^2.3^3$	-108	1	$\equiv 4,\ \frac{\theta^2}{2},\ \theta,\ 1$	$\frac{\theta^2-2}{2}$	$22:2\ =\left(\frac{\theta^2}{2}\right)^3\ 3=(\theta+1)^3(-2\theta^2+3\theta-5)^*$
23	$x^3+x+4=0$	$-2^2.109$	$-2^2.109$	-436	1	......	$21\theta^2-29\theta+61$	$23\begin{cases}2\ =(-\theta-1)^2(2\theta^2-3\theta+6)\ \ 3=(\theta^2-\theta+3)(-\theta^2-\theta+1)\ \ 5=5\\ 7\ =(2\theta+3)(4\theta^2-6\theta+13)\ \ 11=11\ \ 13=(\theta^2-\theta+1)(\theta^2-3\theta-3)\\ 17=(2\theta^2-2\theta+5)(-2\theta^2-6\theta-3)\end{cases}$
24	$x^3-x+4=0$	$-2^2.107$	-107	-107	1	$\frac{\theta^2+\theta}{2},\ \theta,\ 1$	$5\theta^2-9\theta+11\pm k^2$ $\theta^2-\theta-5,\ \frac{\theta^2-\theta+2}{2}$	$24\begin{cases}2\ =(\theta+2)(\theta^2-2\theta+3)\ \ 3=3\ \ 5=(2\theta^2-4\theta+5)(2\theta^2+4\theta+1)\\ 7\ =(2\theta^2+2\theta-3)(-6\theta^2+10\theta-13)\ \ 11=11\ \ 13=13\end{cases}$
25	$x^3+2x+4=0$	$-2^4.29$	$-2^2.29$	-116	1	$\frac{\theta^2}{2},\ \theta,\ 1$	$\theta+1$	$25\begin{cases}2\ =\left(\frac{\theta^2+2}{2}\right)^2\left(\frac{\theta^2}{2}\right)\ \ 3=3\ \ 5=5\ \ 7=(-\theta+1)(\theta^2+\theta+3)\ \ 11=11\ \ 13=13\\ 17=(\theta^2+1)(\theta^2-4\theta+1)\ \ 19=(-2\theta-3)(\theta^2+3)(-\theta^2+2\theta-3)\end{cases}$

PART I.—Continued.

		$d(\theta)$	Δ		h	Basis.	Units.	Factorization of Rational Primes.
26	$x^3-2x+4=0$	red.						26 :
27	$x^3+3x+4=0$	red.						27 :
28	$x^3-3x+4=0$	$-2^2\,.\,3^4$	$-2^2\,.\,3^4$	-324	1	......	$\theta^2-\theta-7$	28 $\begin{cases} 2=(-\theta+1)^2(-\theta-2)\ \ 3=(-\theta^2-\theta+3)^3(-5\theta^2+11\theta-9)^*\ \ 5=5 \\ 7=(2\theta^2+2\theta-5)(2\theta^2-6\theta+5)\ \ 11=(\theta^2+\theta-1)(-\theta^2-3\theta+5)\ \ 13=13 \\ 17=17 \end{cases}$
29	$x^3+4x+4=0$	$-2^4\,.\,43$	$-2^2\,.\,43$	-172	1	$\frac{\theta^2}{2}, \theta, 1$	$\theta+1,\ \theta^2-\theta+5$	29 $\begin{cases} 2=\left(\frac{\theta^2+4}{2}\right)^3(\theta+1)^*\ \ 3=\left(\frac{-\theta^2-2}{2}\right)\left(\frac{-\theta^2+2\theta-2}{2}\right) \\ 5=\left(\frac{\theta^2+6}{2}\right)\left(\frac{-\theta^2-2\theta+2}{2}\right)\ \ 7=\left(\frac{-\theta^2+2}{2}\right)\left(\frac{3\theta^2-2\theta+18}{2}\right)\ \ 11=11 \\ 13=13\ \ 17=17 \end{cases}$
30	$x^3-4x+4=0$	$-2^4\,.\,11$	$-2^2\,.\,11$	-44	1	$\frac{\theta^2}{2}, \theta, 1$	$\theta-1$	30 $\begin{cases} 2=\left(\frac{\theta^2}{2}\right)^3(-\theta^2-2\theta+1)^*\ \ 3=3\ \ 5=5\ \ 7=(-\theta-1)(\theta^2-\theta-3) \\ 11=\theta^2-5)^2(2\theta^2-4\theta+3)\ \ 13=(\theta^2-3)(-\theta^2-4\theta+1) \end{cases}$
31	$x^3+4x+1=0$	-283	-283	-283	2	......	$\theta\pm k^2,\ 4\theta+1$	31 $\begin{cases} (2)=(2,\theta+1)(2,\theta^2+\theta+1)\ \ (3)=(3,\theta-1)(3,\theta^2+\theta-1) \\ (5)=(5,\theta+2)(5,\theta^2-2\theta+3)\ \ (7)=(7)\ \ (11)=(11)\ \ (13)=(13) \end{cases}$
32	$x^3-4x+1=0$	229	229	229	1	......	$\theta,\ \theta+2,\ \theta-2$	32 $\begin{cases} 2=(\theta-1)(\theta^2+\theta-3)\ \ 3=3\ \ 5=5\ \ 7=(2\theta-1)(4\theta^2+2\theta-15) \\ 11=11\ \ 13=(2\theta-3)(4\theta^2+6\theta-7) \end{cases}$
33	$x^3+4x+2=0$	$-2^2.7.13$	$-2^2.7.13$	-364	1	......	$2\theta+1$	33 $\begin{cases} 2=(-\theta)^3(4\theta^2-2\theta+17)^*\ \ 3=(\theta+1)(\theta^2-\theta+5)\ \ 5=5 \\ 7=(-\theta+1)^2(\theta^2+3)\ \ 11=(15\theta^2-7\theta+63)(4\theta^2-9\theta-5) \\ 13=(3\theta^2-\theta+13)^2(\theta^2-6\theta-3)\ \ 17=(-2\theta^2+\theta-7)(-3\theta^2+\theta-1) \end{cases}$
34	$x^3-4x+2=0$	$2^2\,.\,37$	$2^2.\,37$	148	1	......	$\theta-1,\ -2\theta+1$	34 : $2=(-\theta)^3(4\theta^2+2\theta-15)^*\ \ 3=3\ \ 5=(-\theta-1)(\theta^2-\theta-3)\ \ 7=7\ \ 11=11$
35	$x^3+4x+3=0$	-499	-499	-499	1	......	$3\theta+2$	35 : $2=(\theta+1)(\theta^2-\theta+5)\ \ 3=(-\theta)(\theta^2+4)\ \ 5=5$
36	$x^3-4x+3=0$	red.						36 :
37	$x^3+5=0$	$-3^3\,.\,5^2$	$-3^3\,.\,5^2$	-675	1	......	$2\theta^2+4\theta+1$	37 $\begin{cases} 2=(\theta^2-2\theta+3)(\theta^2+\theta-1)\ \ 3=(\theta+2)^3(14\theta^2-24\theta+41)^*\ \ 5=(-\theta)^3 \\ 7=7\ \ 11=(2\theta^2-3\theta+6)(-3\theta^2-2\theta+6)\ \ 13=(-\theta+2)(\theta^2+2\theta+4) \\ 17=(\theta^2-2)(2\theta^2-5\theta+4)\ \ 19=19\ \ 23=(2\theta^2-4\theta+7)(2\theta^2+8\theta+9) \end{cases}$
38	$x^3+x+5=0$	$-7\,.\,97$	$-7\,.\,97$	-679	1	......	$-4\theta^2+6\theta-13$	38 $\begin{cases} 2=2\ \ 3=(-\theta-1)(\theta^2-\theta+2)\ \ 5=(-\theta)(\theta+2)(\theta^2-\theta+3) \\ 7=(-\theta^2+2\theta-4)^2(2\theta^2+\theta-3)\ \ 11=11\ \ 13=13\ \ 17=17\ \ 19=19 \\ 23=(\theta^2-3)(-4\theta^2+5\theta-16) \end{cases}$
39	$x^3-x+5=0$	$-11\,.\,61$	$-11\,.\,61$	-671	1	......	$\theta+2$	39 $\begin{cases} 2=2\ \ 3=3\ \ 5=(-\theta)(\theta+1)(\theta-1)\ \ 7=7\ \ 11=(-\theta+2)^2(\theta^2+\theta-1) \\ 13=(\theta^2-3)(2\theta^2-5\theta+4)\ \ 17=17\ \ 19=(\theta+3)(\theta^2-3\theta+8) \\ 23=(\theta^2-2)(\theta^2-5\theta+1) \end{cases}$
40	$x^3+2x+5=0$	$-7\,.\,101$	$-7\,.\,101$	-707	1	......	$3\theta+4$	40 $\begin{cases} 2=(-\theta-1)(\theta^2-\theta+3)\ \ 3=3\ \ 5=(-\theta)(\theta^2+2) \\ 7=(\theta+2)^2(2\theta^2-3\theta+8)\ \ 11=(2\theta+3)(4\theta^2-6\theta+17) \\ 13=(\theta^2-2\theta+4)(2\theta+3\theta+2)\ \ 17=(-\theta+2)(\theta^2+2\theta+6) \\ 19=(2\theta^2-2\theta+7)(-2\theta^2-6\theta-3)\ \ 23=23 \end{cases}$

PART I.—Continued.

		$d(\theta)$	Δ		h	Basis.	Units.	Factorization of Rational Primes.
41	$x^3-2x+5=0$	-643	-643	-643	2	......	$\theta+2\pm k^2$	41 : $(2)=(2,\theta+1)(2,\theta^2+\theta+1)$ $(3)=(3,\theta+1)(3,\theta^2-\theta-1)$ $(5)=(-\theta)(\theta^2-2)$ $(7)=(7)$
42	$x^3+3x+5=0$	$-3^3\,.\,29$	$-3\,.\,29$	-87	1	$\frac{\theta^2+\theta+1}{3},\ \theta,\ 1$	$\theta+1,\ \frac{\theta^2+\theta+1}{3}$	42 : $2=2$ $3=\left(\frac{\theta^2-2\theta-2}{3}\right)^2\left(\frac{4\theta^2-5\theta+19}{3}\right)$ $5=(-\theta)(\theta^2+3)$ $7=7$
43	$x^3-3x+5=0$	$-3^4\,.\,7$	$-3^4\,.\,7$	-567	1	......	$\theta^2-2\theta+2$	43 $\begin{cases} 2=2 \ \ 3=(-\theta+1)^3(\theta^2+\theta-3)^*\ \ 5=(-\theta)(\theta^2-3) \\ 7=(-\theta-1)^2(-\theta+2)\ \ 11=11\ \ 13=(-\theta-3)(-\theta^2+3\theta-6) \\ 17=(-\theta^2-\theta+4)(2\theta^2-\theta+3)\ \ 19=(\theta^2+\theta-1)(-\theta^2-4\theta+6) \\ 23=(-\theta+3)(\theta^2-2)(-\theta^2-2\theta+2) \end{cases}$
44	$x^3+4x+5=0$	red.						44 :
45	$x^3-4x+5=0$	-419	-419	-419	1	......	$2\theta^2-5\theta+4$	45 $\begin{cases} 2=(-\theta+1)(\theta^2+\theta-3)\ \ 3=3\ \ 5=(-\theta)(\theta+2)(\theta-2)\ \ 7=7 \\ 11=(-\theta^2-2\theta+2)(2\theta^2-\theta-2)\ \ 13=13\ \ 17=(\theta^2-2)(-2\theta^2-5\theta+4) \end{cases}$
46	$x^3+5x+5=0$	$-5^2\,.\,47$	$-5^2\,.\,47$	-1175	1	......	$\theta+1$	46 $\begin{cases} 2=2\ \ 3=3\ \ 5=(-\theta)^3(\theta^2-\theta+6)^*\ \ 7=7\ \ 11=(-\theta+1)(\theta^2+\theta+6) \\ 13=(\theta+2)(\theta^2-2\theta+9)\ \ 17=17\ \ 19=(-2\theta-1)(4\theta^2-2\theta+21) \\ 23=(-\theta+2)(\theta^2+2\theta+9)\ \ 29=(\theta^2+4)(\theta^2-5\theta+1) \\ 31=(\theta^2+\theta+1)(5\theta^2-6\theta+26) \end{cases}$
47	$x^3-5x+5=0$	$-5^2\,.\,7$	$-5^2\,.\,7$	-175	1	......	$\theta-1$	47 $\begin{cases} 2=2\ \ 3=(-\theta+2)(\theta^2+2\theta-1)\ \ 5=(-\theta)^3(\theta^2+\theta-4)^* \\ 7=(-\theta-2)^2(\theta^2-3\theta+3)\ \ 11=11 \end{cases}$
48	$x^3+5x+1=0$	$-17\,.\,31$	$-17\,.\,31$	-527	1	......	θ	48 : $2=2$ $3=3$ $5=(\theta+1)(\theta^2-\theta+6)$
49	$x^3-5x+1=0$	$11\,.\,43$	$11\,.\,43$	473	1	......	θ	49 : $2=2$ $3=(\theta-1)(\theta^2+\theta-4)$
50	$x^3+5x+2=0$	$-2^5\,.\,19$	$-2^3\,.\,19$	-152	1	$\frac{\theta^2+\theta}{2},\ \theta,\ 1$	......	50 : $2=\left(\frac{\theta^2-\theta+4}{2}\right)^2(-\theta)$ $3=3$
51	$x^3-5x+2=0$	red.						51 :
52	$x^3+5x+3=0$	-743	-743	-743	1	......	......	52 $\begin{cases} 2=2\ \ 3=(-\theta)(\theta+1)(2\theta^2-\theta+11)\ \ 5=(2\theta^2-\theta+10)(\theta^2-2\theta-1) \\ 7=(\theta^2-\theta+5)(\theta^2+2\theta+2) \end{cases}$
53	$x^3-5x+3=0$	257	257	257	1	......	$\theta-1$	53 : $2=2$ $3=(-\theta)(\theta^2-5)$
54	$x^3+5x+4=0$	$-2^2\,.\,233$	$-2^2\,.\,233$	-932	1	......	......	54 $\begin{cases} 2=(\theta+1)^2(4\theta^2-3\theta+22)\ \ 3=3\ \ 5=(\theta^2-\theta+5)(\theta^2+\theta+1) \\ 7=(2\theta^2-1)(22\theta^2-16\theta+121) \end{cases}$
55	$x^3-5x+4=0$	red.						55 :
56	$x^3+6=0$	$-2^2\,.\,3^5$	$-2^2\,.\,3^5$	-972	1	......	$3\theta^2+6\theta+1$	56 $\begin{cases} 2=(\theta+2)^3(33\theta^2-60\theta+109)^*\ \ 3=(\theta^2-2\theta+3)^3(3\theta^2+6\theta+1)^* \\ 5=(-\theta-1)(\theta^2-\theta+1)\ \ 7=(-\theta+1)(2\theta^2-4\theta+7)(\theta^2-\theta-5) \end{cases}$
57	$x^3+x+6=0$	$-2^4\,.\,61$	$-2^2\,.\,61$	-244	1	$\frac{\theta^2+\theta}{2},\ \theta,\ 1$	......	57 : $2=\left(\frac{\theta^2-\theta+4}{2}\right)^2\left(\frac{-\theta^2-3\theta-2}{2}\right)$ $3=\left(\frac{3\theta^2-5\theta+12}{2}\right)\left(\frac{-\theta^2+3\theta+8}{2}\right)$
58	$x^3-x+6=0$	red.						58 :
59	$x^3+2x+6=0$	$-2^2\,.\,251$	$-2^2\,.\,251$	-1004	1	......	$7\theta^2+15\theta+7$	59 : $2=(2\theta^2-3\theta+8)^3(7\theta^2+15\theta+7)^*$ $3=(-\theta-1)(2\theta+3)(9\theta^2-13\theta+37)$
60	$x^3-2x+6=0$	$-2^2.5.47$	$-2^2.5.47$	-940	1	......	$3\theta^2+7\theta+1$	60 $\begin{cases} 2=(-\theta-2)^3(28\theta^2-61\theta+77)^*\ \ 3=(\theta^2-2\theta+3)(-\theta^2+5) \\ 5=(2\theta^2-4\theta+5)^2(4\theta^2-19)\ \ 7=(-\theta-1)(\theta^2-\theta-1) \end{cases}$

PART I.—Continued.

		$d(\theta)$	Δ		h	Basis.	Units.	Factorization of Rational Primes.
61	$x^3+3x+6=0$	$-2^3.3^3.5$	$-2^3.3^3.5$	-1080	1	......	......	61 $\begin{cases} 2 = (-\theta-1)^2(3\theta^2-4\theta+14) \;\; 3=(-\theta^2+\theta+3)^3(2473\theta^2-3185\theta+11521)^* \\ 5 = (\theta^2-\theta+5)(-\theta^2-\theta+1) \;\; 7=(2\theta^2+8\theta+7)(62\theta^2-80\theta+289) \end{cases}$
62	$x^3-3x+6=0$	$-2^5.3^3$	$-2^3.3^3$	-216	1	$\frac{\theta^2+\theta}{2}, \theta, 1$	$\frac{-\theta^2+\theta+8}{2}$	62 : $2 = \left(\frac{-\theta^2-\theta+4}{2}\right)^2(\theta^2-2\theta+2) \;\; 3=\left(\frac{\theta^2-\theta}{2}\right)^3\left(\frac{-\theta^2+\theta+8}{2}\right)^*$
63	$x^3+4x+6=0$	$-2^2.307$	$-2^2.307$	-1228	3	......	$\theta+1\mp k^3$	63 : $(2)=(2,\theta)^3 \;\; (3)=(3,\theta)(3,\theta^2+4) \;\; (5)=(5,\theta+2)(5,\theta^2-2\theta+8) \;\; (7)=(7)$
64	$x^3-4x+6=0$	$-2^2.179$	$-2^2.179$	-716	1	......	$40\theta^2-101\theta+95$	64 $\begin{cases} 2 = (-\theta^2-\theta+4)^3(40\theta^2-101\theta+95)^* \;\; 3=(-\theta+1)(\theta^2-3\theta+3)(-2\theta-5) \\ 5 = 5 \;\; 7=(\theta^2+2\theta-1)(\theta^2-4\theta+5) \end{cases}$
65	$x^3+5x+6=0$	red.						65 :
66	$x^3-5x+6=0$	$-2^3.59$	$-2^3.59$	-472	1	......	......	66 $\begin{cases} 2 = (-\theta+1)^2(-\theta^2-2\theta+2) \;\; 3=(\theta^2-3\theta+3)(\theta^2+3\theta+1) \\ 5 = (-\theta^2-\theta+5)(\theta^2-\theta+1) \end{cases}$
67	$x^3+6x+6=0$	$-2^2.3^3.17$	$-2^2.3^3.17$	-1836	3	......	$\theta+1\mp k^3$	67 $\begin{cases} (2) = (2,\theta)^3 \;\; (3)=(3,\theta)^3 \;\; (5)=(5) \;\; (7)=(7,\theta+2)(7,\theta^2-2\theta+3) \\ (11)=(11) \end{cases}$
68	$x^3-6x+6=0$	$-2^2.3^3$	$-2^2.3^3$	-108	1	$\equiv 4$	......	68 :
69	$x^3+6x+1=0$	$-3^4.11$	$-3^4.11$	-891	3	......	$\theta\mp k^3 \;\; 6\theta+1$	69 $\begin{cases} (2)=(2,\theta+1)(2,\theta^2+\theta+1) \;\; (3)=(3,\theta+1)^3 \;\; (5)=(5) \\ (7)=(7,\theta-2)(7,\theta^2+2\theta+3) \end{cases}$
70	$x^3-6x+1=0$	$3^3.31$	$3^3.31$	837	1	......	θ	70 $\begin{cases} 2 = (-\theta^2-2\theta+1)(-\theta^2+\theta+3) \;\; 3=(\theta-2)^3(6\theta^2+14\theta-3)^* \\ 5 = (-\theta-2)(\theta^2-2\theta-2) \end{cases}$
71	$x^3+6x+2=0$	$-2^2.3^5$	$-2^2.3^3$	-108	1	$\equiv 4, \frac{\theta^2+\theta+1}{3},$	......	71 :
72	$x^3-6x+2=0$	$2^2.3^3.7$	$2^2.3^3.7$	756	1	$\theta-1, 1$	$3\theta-1$	72 : $2 = (-\theta)^3(9\theta^2+3\theta-53)^* \;\; 3=(\theta-1)^3(2\theta^2+\theta-11)^* \;\; 5=5$
73	$x^3+6x+3=0$	$-3^3.41$	$-3^3.41$	-1107	2	......	$2\theta+1\mp k^2$	73 $\begin{cases} (2)=(2,\theta+1)(2,\theta^2+\theta+1) \;\; (3)=(-\theta)^3 \;\; (5)=(5,\theta-1)(5,\theta^2+\theta+2) \\ (7)=(7,\theta+3)(7,\theta^2-3\theta+1) \end{cases}$
74	$x^3-6x+3=0$	$3^3.23$	$3^3.23$	621	1	......	$\theta-2$	74 : $2 = (\theta-1)(\theta^2+\theta-5) \;\; 3=(-\theta)^3(4\theta^2+2\theta-23)^* \;\; 5=5$
75	$x^3+6x+4=0$	$-2^4.3^4$	$-2^2.3^4$	-324	1	$\frac{\theta^2}{2}, \theta, 1$	$\theta^2-\theta-1$	75 : $2 = \left(\frac{\theta^2+6}{2}\right)^2\left(\frac{\theta^2}{2}\right) \;\; 3=(\theta+1)^3(8\theta^2-5\theta+51)^* \;\; 5=5$
76	$x^3-6x+4=0$	red.						76 :
77	$x^3+6x+5=0$	$-3^4.19$	$-3^4.19$	-1539	1	......	$2\theta^2-34\theta-27$	77 $\begin{cases} 2 = (\theta+1)(\theta^2-\theta+7) \;\; 3=(4\theta^2-3\theta+26)^3(2\theta^2-34\theta-27)^* \\ 5 = (-\theta)(21\theta^2-16\theta+138)(-4\theta-3) \;\; 7=7 \;\; 11=11 \end{cases}$
78	$x^3-6x+5=0$	red.						78 :
79	$x^3+7=0$	$-3^3.7^2$	$-3^3.7^2$	-1323	3	......	$\theta+2\mp k^3$	79 $\begin{cases} (2)=(2,\theta+1)(2,\theta^2+\theta+1) \;\; (3)=(3,\theta+1)^3 \\ (5)=(5,\theta-2)(5,\theta^2+2\theta+4) \;\; (7)=(-\theta)^3 \end{cases}$
80	$x^3+x+7=0$	-1327	-1327	-1327	1	......	......	80 : $2 = 2 \;\; 3=(\theta+2)(\theta^2-2\theta+5) \;\; 5=(-\theta-1)(\theta^2-\theta+2) \;\; 7=(-\theta)(\theta^2+1$
81	$x^3-x+7=0$	-1319	-1319	-1319	1	......	$\theta+2$	81 : $2 = 2 \;\; 3=3 \;\; 5=5 \;\; 7=(-\theta)(-\theta+1)(-\theta-1)$
82	$x^3+2x+7=0$	-5.271	-5.271	-1355	1	......	......	82 $\begin{cases} 2 = (-2\theta^2+3\theta-9)(\theta^2+\theta-1) \;\; 3=3 \;\; 5=(2\theta+3)^2(28\theta^2-44\theta+125) \\ 7 = (-\theta)(\theta^2+2) \end{cases}$
83	$x^3-2x+7=0$	-1291	-1291	-1291	1	......	......	83 $\begin{cases} 2 = (\theta^2-2\theta+3)(-\theta^2-\theta+3) \;\; 3=(-\theta-2)(\theta^2-2\theta+2) \;\; 5=5 \\ 7 = (-\theta)(-2\theta^2-\theta+8)(9\theta^2-20\theta+28) \end{cases}$

PART I.—Continued.

		$d(\theta)$	Δ		h	Basis.	Units.	Factorization of Rational Primes.
84	$x^3+3x+7=0$	$-3^3.53$	$-3^3.53$	-1431	1	.....	$-\theta^2+2$	$84: 2\ =2\ \ 3=(-\theta-1)^3(5\theta^2-7\theta+25)^*\ \ 5=5\ \ 7=(-\theta)(\theta+2)(\theta^2-\theta+5)$
85	$x^3-3x+7=0$	$-3^5.5$	$-3^3.5$	-135	1	$\frac{\theta^2-\theta+1}{3},\ \theta,\ 1$	$\frac{\theta^2-\theta+1}{3}$	$85: 2\ =2\ \ 3=\left(\frac{\theta^2+2\theta+1}{3}\right)^3\left(\frac{\theta^2-\theta-8}{3}\right)^*$
86	$x^3+4x+7=0$	-1579	-1579	-1579	1	......	......	$86\begin{cases}2\ =(-\theta-1)(\theta^2-\theta+5)\ \ 3=(4\theta^2-5\theta+22)(\theta^2-2\theta-4)\ \ 5=5\\ 7\ =(-\theta)(\theta^2+4)\ \ 11=11\end{cases}$
87	$x^3-4x+7=0$	-11.97	-11.97	-1067	1	......	......	$87\begin{cases}2\ =(-\theta^2-\theta+4)(-\theta^2+3\theta-3)\ \ 3=3\ \ 5=(-\theta^2+2\theta-2)(2\theta^2+3\theta-6)\\ 7\ =(-\theta)(-\theta+2)(-\theta-2)\end{cases}$
88	$x^3+5x+7=0$	-1823	-1823	-1823	2	......	$\theta+1\mp k^2$	$88\begin{cases}(2)=(2)\ \ (3)=(3)\ \ (5)=(5,\ \theta-2)(5,\ \theta^2+2\theta-1)\\ (7)=(-\theta)(7,\ \theta-3)(7,\ \theta+3)\ \ (11)=(\theta+2)(11,\ \theta+4)(11,\ \theta+5)\end{cases}$
89	$x^3-5x+7=0$	-823	-823	-823	1	......	......	$89\begin{cases}2\ =2\ \ 3=(-\theta+1)(\theta^2+\theta-4)\ \ 5=(-\theta+2)(\theta^2+2\theta-1)\\ 7\ =(-\theta)(\theta^2-5)\end{cases}$
90	$x^3+6x+7=0$	red.						$90:$
91	$x^3-6x+7=0$	$-3^3.17$	$-3^3.17$	-459	1	......	$2\theta^2+2\theta-11$	$91\begin{cases}2\ =(-\theta+1)(\theta^2+\theta-5)\ \ 3=(-\theta+2)^3(2\theta^2+2\theta-11)^*\ \ 5=5\\ 7\ =(-\theta)(\theta^2-6)\ \ 11=(-\theta-2)(2\theta^2+4\theta-5)(3\theta^2-8\theta+6)\ \ 13=13\end{cases}$
92	$x^3+7x+7=0$	$-5.7^2.11$	$-5.7^2.11$	-2695	3	......	$\theta+1\mp k^3$	$92\begin{cases}(2)=(2)\ \ (3)=(3,\ \theta-1)(3,\ \theta^2+\theta-1)\ \ (5)=(5,\ \theta-1)^2\ (5,\ \theta+2)\\ (7)=(-\theta)^3\ \ (11)=(11,\ \theta-4)^2\ (11,\ \theta-3)\ \ (13)=(13)\end{cases}$
93	$x^3-7x+7=0$	7^2	7^2	49	1	......	......	$93:$
94	$x^3+7x+1=0$	-1399	-1399	-1399	2	......	$\theta\mp k^2$	$94: (2)=(2)\ \ (3)=(3,\ \theta-1)(3,\ \theta^2+\theta+2)\ \ (5)=(5)\ \ (7)=(\theta+1)(7,\ \theta+2)(7,\ \theta-3)$
95	$x^3-7x+1=0$	5.269	5.269	1345	1	.....	θ	$95: 2\ =2\ \ 3=3\ \ 5=(\theta-2)^2(\theta^2+3\theta+1)\ \ 7=(-\theta-1)(-\theta-2)(-\theta+3)$
96	$x^3+7x+2=0$	$-2^3.5.37$	$-2^3.5.37$	-1480	1	......	......	$96\begin{cases}2\ =(2\theta^2-3\theta-1)^2(5501\theta^2-1554\theta+38946)\ \ 3=(14\theta^2-4\theta+99)(2\theta^2+4\theta+1)\\ 5\ =(25\theta^2-7\theta+177)^2(24\theta^2-18\theta-7)\ \ 7=7\end{cases}$
97	$x^3-7x+2=0$	$2^4.79$	$2^2.79$	316	1	$\frac{\theta^2+\theta}{2},\ \theta,\ 1$	$\frac{\theta^2-\theta}{2},\ 3\theta^2+\theta-2$	$97: 2\ =\left(\frac{\theta^2+\theta}{2}\right)^2\left(\frac{-17\theta^2-5\theta+118}{2}\right)\ \ 3=3$
98	$x^3+7x+3=0$	$-5.17.19$	$-5.17.19$	-1615	1	......	......	$98\begin{cases}2\ =2\ \ 3=(-\theta)(\theta^2+7)\ \ 5=(\theta+1)^2(2\theta^2-\theta+14)\\ 7\ =7\ \ 11=(-\theta+1)(\theta^2+\theta+8)\end{cases}$
99	$x^3-7x+3=0$	1129	1129	1129	1	......	θ^2-8	$99: 2\ =2\ \ 3=(-\theta)(\theta-1)(-2\theta^2-\theta+13)\ \ 5=5\ \ 7=7$
100	$x^3+7x+4=0$	$-2^2.11.41$	-11.41	-451	1	$\frac{\theta^2+\theta}{2},\ \theta,\ 1$	......	$100: 2\ =\left(\frac{\theta^2-\theta+8}{2}\right)(\theta+1)\ \ 3=\left(\frac{\theta^2-\theta+6}{2}\right)\left(\frac{\theta^2+\theta+2}{2}\right)\ \ 5=5$
101	$x^3-7x+4=0$	$2^2.5.47$	$2^2.5.47$	940	1	......	$-2\theta^2+6\theta-3$	$101: 2\ =(\theta-1)^2(-2\theta^2-\theta+14)\ \ 3=3\ \ 5=(2\theta-1)^2(-20\theta^2-12\theta+133)$
102	$x^3+7x+5=0$	-23.89	-23.89	-2047	1	......	......	$102\begin{cases}2\ =2\ \ 3=(\theta+1)(\theta^2-\theta+8)\ \ 5=(-\theta)(\theta^2+7)\ \ 7=7\\ 11=(3\theta^2-2\theta+23)(-2\theta^2+\theta+2)\end{cases}$
103	$x^3-7x+5=0$	17.41	17.41	697	1	......	$\theta-2$	$103: 2\ =2\ \ 3=3\ \ 5=(-\theta)(\theta^2-7)$
104	$x^3+7x+6=0$	$-2^3.293$	$-2^3.293$	-2344	1	......	......	$104\begin{cases}2\ =(\theta+1)^2(5\theta^2-4\theta+38)\ \ 3=(-\theta^2+3\theta+3)(19\theta^2-15\theta+145)\ \ 5=5\\ 7\ =(\theta^2-\theta+7)(7\theta^2-11\theta-13)\ \ 11=11\ \ 13=13\end{cases}$
105	$x^3-7x+6=0$	red.						$105:$
106	$x^3+8=0$	red.						$106:$

PART I.—Continued.

		$d(\theta)$	Δ		h	Basis.	Units.	Factorization of Rational Primes.
107	$x^3+x+8=0$	$-2^2 \,.\, 433$	$-2^2 \,.\, 433$	-1732	2	......	$6\theta+11 \pm k^2$	107 $\begin{cases} (2)=(\theta+2)(2,\theta-1)^2 \;\; (3)=(3,\theta+1)(3,\theta^2-\theta+2) \\ (5)=(5,\theta-1)(5,\theta^2+\theta+2) \;\; (7)=(7) \end{cases}$
108	$x^3-x+8=0$	$-2^2 \,.\, 431$	-431	-431	1	$\frac{\theta^2+\theta}{2},\theta,1$	$6\theta-11$	108 : $2=(-\theta-2)(\theta^2-2\theta+3) \;\; 3=3 \;\; 5=5$
109	$x^3+2x+8=0$	$-2^5 \,.\, 5 \,.\, 11$	$-2^3 \,.\, 5 \,.\, 11$	-440	1	$\frac{\theta^2}{2},\theta,1$	......	109 : $2=\left(\frac{\theta^2-2\theta-6}{2}\right)^2\left(\frac{\theta^2+4\theta+4}{2}\right) \;\; 3=3 \;\; 5=(-\theta-1)^2(\theta^2-2\theta+5)$
110	$x^3-2x+8=0$	$-2^5 \,.\, 53$	$-2^3 \,.\, 53$	-424	1	$\frac{\theta^2}{2},\theta,1$	......	110 : $2=\left(\frac{\theta^2-2\theta+2}{2}\right)^2\left(\frac{\theta^2+4\theta+4}{2}\right) \;\; 3=(\theta^2-2\theta+3)(-\theta^2-2\theta+1) \;\; 5=5$
111	$x^3+3x+8=0$	$-2^2 \,.\, 3^3 \,.\, 17$	$-3^3 \,.\, 17$	-459	1	$\frac{\theta^2+\theta}{2},\theta,1$	$2\theta+3$	111 : $2=\left(\frac{\theta^2-\theta+4}{2}\right)(-\theta-1) \;\; 3=\left(\frac{\theta^2-\theta+6}{2}\right)^3(-2\theta-3)^* \;\; 5=5$
112	$x^3-3x+8=0$	$-2^2 \,.\, 3^4 \,.\, 5$	$-2^2 \,.\, 3^4 \,.\, 5$	-1620	3	......	$2\theta+5 \pm k^3$	112 $\begin{cases} (2)=(2,\theta-1)^2(2,\theta+2) \;\; (3)=(3,\theta-1)^3 \;\; (5)=(5,\theta+1)^2(5,\theta-2) \\ (7)=(7) \;\; (11)=(11,\theta+4)(11,\theta^2-4\theta+2) \end{cases}$
113	$x^3+4x+8=0$	$-2^6 \,.\, 31$	-31	-31	1	$\equiv 2 \,.\, \frac{\theta^2}{4},\frac{\theta}{2},1$	......	113 :
114	$x^3-4x+8=0$	$-2^6 \,.\, 23$	-23	-23	1	$\equiv 3 \,.\, \frac{\theta^2}{4},\frac{\theta}{2},1$	......	114 :
115	$x^3+5x+8=0$	$-2^2 \,.\, 557$	$-2^2 \,.\, 557$	-2228	1	......	......	115 $\begin{cases} 2=(-\theta-1)^2(4\theta^2-5\theta+26) \;\; 3=3 \;\; 5=(2\theta^2-3)(26\theta^2-32\theta+169) \\ 7=(\theta^2-\theta+7)(-\theta^2-\theta+1) \;\; 11=11 \;\; 13=(4\theta+5)(16\theta^2-20\theta+105) \end{cases}$
116	$x^3-5x+8=0$	$-2^2 \,.\, 307$	-307	-307	1	$\frac{\theta^2+\theta}{2},\theta,1$	......	116 : $2=\left(\frac{\theta^2+\theta-4}{2}\right)(-\theta+1) \;\; 3=\left(\frac{-\theta^2-\theta+6}{2}\right)\left(\frac{\theta^2-\theta+2}{2}\right)$
117	$x^3+6x+8=0$	$-2^5 \,.\, 3^4$	$-2^3 \,.\, 3^4$	-648	3	$\frac{\theta^2}{2},\theta,1$	$\theta+1 \pm k^3$	117 $\begin{cases} (2)=\left(2,\frac{\theta^2+2\theta+2}{2}\right)^2\left(2,\frac{\theta^2}{2}\right) \;\; (3)=(3,\theta-1)^3 \\ (5)=(5,\theta-1)(5,\theta^2+\theta+2) \;\; (7)=(7,\theta-2)(7,\theta^2+2\theta+3) \end{cases}$
118	$x^3-6x+8=0$	$-2^5 \,.\, 3^3$	$-2^3 \,.\, 3^3$	-216	1	$\frac{\theta^2}{2},\theta,1$	$\theta+3$	118 : $2=\left(\frac{\theta^2+2\theta-2}{2}\right)^2\left(\frac{\theta^2-4\theta+4}{2}\right) \;\; 3=(-\theta+1)^3(\theta+3)^*$
119	$x^3+7x+8=0$	red.						119 :
120	$x^3-7x+8=0$	$-2^2 \,.\, 89$	$-2^2 \,.\, 89$	-356	1	......	......	120 : $2=(-\theta+1)^2(-2\theta^2-3\theta+10) \;\; 3=3 \;\; 5=5$
121	$x^3+8x+8=0$	$-2^6 \,.\, 59$	-59	-59	1	$\equiv 9 \,.\, \frac{\theta^2}{4},\frac{\theta}{2},1$	......	121 :
122	$x^3-8x+8=0$	red.						122 :
123	$x^3+8x+1=0$	$-5^2 \,.\, 83$	-83	-83	1	$\frac{\theta^2+2\theta+2}{5},\theta,1$	θ	123 : $2=\left(\frac{\theta^2+2\theta+7}{5}\right)\left(\frac{\theta^2-3\theta+7}{5}\right)$
124	$x^3-8x+1=0$	$43 \,.\, 47$	$43 \,.\, 47$	2021	1	......	θ	124 $\begin{cases} 2=(\theta+3)(\theta^2-3\theta+1) \;\; 3=(-2\theta^2+6\theta-1)(2\theta^2+2\theta-11) \;\; 5=5 \\ 7=(\theta-2)(\theta^2+2\theta-4) \end{cases}$
125	$x^3+8x+2=0$	$-2^2 \,.\, 7^2 \,.\, 11$	$-2^2 \,.\, 11$	-44	1	$\frac{\theta^2-3\theta+3}{7},\theta,1$	......	125 :
126	$x^3-8x+2=0$	$2^2 \,.\, 5 \,.\, 97$	$2^2 \,.\, 5 \,.\, 97$	1940	1	......	$\theta^2-3\theta+1$	126 $\begin{cases} 2=(-\theta)^3(16\theta^2+4\theta-127)^* \;\; 3=(\theta^2-7)(\theta^2+2\theta-1) \\ 5=(\theta-1)^2(-\theta^2+9) \;\; 7= \end{cases}$

PART I.—Continued.

		$d(\theta)$	Δ		h	Basis.	Units.	Factorization of Rational Primes.
127	$x^3+3x+3=0$	-29.79	-29.79	-2291	1	......	......	$127\begin{cases}2=(8\theta^2-3\theta+65)(\theta^2+3\theta+1)\ \ 3=(-\theta)(2\theta^2-2\theta-1)(103\theta^2-38\theta+838)\ \ 5=5\\ 7=(7\theta^2+8\theta+2)(442\theta^2-163\theta+3596)\ \ 11=(2\theta^2-\theta+16)(\theta^2+4\theta+2)\ \ 13=13\end{cases}$
128	$x^3-3x+3=0$	red.						128 :
129	$x^3+8x+4=0$	$-2^4.5.31$	$-2^2.5.31$	-610	1	$\frac{\theta^2}{2},\theta,1$	$2\theta+1$	$129:2=\left(\frac{\theta^2}{2}\right)^3(140\theta^2-68\theta+1153)^*\ \ 3=3\ \ 5=\left(\frac{\theta^2+6}{2}\right)^2(\theta+1)$
130	$x^3-8x+4=0$	$2^4.101$	$2^2.101$	404	1	$\frac{\theta^2}{2},\theta,1$	$2\theta-1$	$130:2=\left(\frac{\theta^2}{2}\right)^3(132\theta^2+68\theta-1023)^*\ \ 3=(\theta-1)(\theta^2+\theta-7)$
131	$x^3+8x+5=0$	-7.389	-7.389	-2723	1	......	......	$131\begin{cases}2=(5\theta^2-3\theta+42)(-\theta^2+\theta+1)\ \ 3=3\ \ 5=(-\theta)(\theta^2+8)\\ 7=(-2\theta-1)^2(20\theta^2-12\theta+167)\ \ 11=11\ \ 13=(2\theta^2-\theta+16)(\theta^2-4\theta^2-2)\end{cases}$
132	$x^3-8x+5=0$	1373	1373	1373	1	......	......	$132:2=(\theta-1)(\theta^2+\theta-7)\ \ 3=(\theta-2)(\theta^2+2\theta-4)\ \ 5=(-\theta)(\theta^2-8)\ \ 7=7$
133	$x^3+8x+6=0$	$-2^2.5.151$	$-2^2.5.151$	-3020	3	......	......	$133\begin{cases}(2)=(2,\theta)^3\ \ (3)=(\theta+1)(3,\theta)(3,\theta-1)\ \ (5)=(5,\theta-2)^2(5,\theta-1)\\ (7)=(7)\ \ (11)=(11)\ \ (13)=(13)\end{cases}$
134	$x^3-8x+6=0$	$2^2.269$	$2^2.269$	1076	1	......	$3\theta^2-10\theta+7$	$134\begin{cases}2=(\theta-2)^3(7\theta^2+16\theta-19)^*\ \ 3=(-\theta-3)(\theta^2-3\theta+1)\ \ 5=5\\ 7=(-2\theta^2-4\theta+7)(-2\theta^2+4\theta+1)\end{cases}$
135	$x^3+8x+7=0$	-3371	-3371	-3371	1	......	......	$135\begin{cases}2=(\theta+1)(\theta^2-\theta+9)\ \ 3=3\ \ 5=5\ \ 7=(-\theta)(\theta^2+8)\\ 11=(6\theta^2-5\theta+52)(\theta^2+8\theta+6)\ \ 13=13\end{cases}$
136	$x^3-8x+7=0$	red.						136 :
137	$x^3+9=0$	-3^7	-3^5	-243	1	$\equiv 11$	......	137 :
138	$x^3+x+9=0$	-7.313	-7.313	-2191	2	......	$\theta-2\pm k^2$	$138\begin{cases}(2)=(2)\ \ (3)=(3,\theta)(3,\theta^2+1)\ \ (5)=(5)\ \ (7)=(\theta+1)(7,\theta+3)^2\\ (11)=(\theta-1)(11,\theta-4)(11,\theta+5)\ \ (13)=(13,\theta-3)(13,\theta^2+3\theta+10)\end{cases}$
139	$x^3-x+9=0$	-37.59	-37.59	-2183	2	......	$\theta^2-5\pm k^2$	$139\begin{cases}(2)=(2)\ \ (3)=(-\theta-2)(3,\theta)(3,\theta+1)\ \ (5)=(5,\theta-2)(5,\theta^2+2\theta+3)\ \ (7)=(7)\\ (11)=(11,\theta-3)(11,\theta^2+3\theta+8)\ \ (13)=(13)\end{cases}$
140	$x^3+2x+9=0$	-7.317	-7.317	-2219	2	......	......	$140\begin{cases}(2)=(2,\theta+1)(2,\theta^2+\theta+1)\ \ (3)=(\theta+2)(3,\theta)(3,\theta+1)\ \ (5)=(5)\\ (7)=(4\theta^2-7\theta+20)(7,\theta-2)^2\ \ (11)=(11)\ \ (13)=(13)\end{cases}$
141	$x^3-2x+9=0$	-5.433	-5.433	-2155	1	......	$16\theta^2-25\theta-152$	$141\begin{cases}2=(-\theta^2-2\theta+1)(3\theta^2-7\theta+11)\ \ 3=(20\theta^2-48\theta+75)(4\theta^2-23)\\ 5=(-\theta-2)^2(2\theta^2-5\theta+8)\ \ 7=7\ \ 11=11\\ 13=(-\theta+2)(2\theta+5)(\theta^2-2\theta+4)\end{cases}$
142	$x^3+3x+9=0$	$-3^3.5.17$	$-3.5.17$	-255	1	$\frac{\theta^2}{3},\theta,1$	......	$142:2=2\ \ 3=\left(\frac{\theta^2+3}{3}\right)^2\left(\frac{\theta^2}{3}\right)$
143	$x^3-3x+9=0$	$-3^3.7.11$	$-3.7.11$	-231	1	$\frac{\theta^2}{3},\theta,1$	......	$143:2=2\ \ 3=\left(\frac{\theta^2-3}{3}\right)^2\left(\frac{\theta^2}{3}\right)$
144	$x^3+4x+9=0$	-7.349	-7.349	-2443	2	......	$\theta^2-4\theta-8$	$144\begin{cases}(2)=(2,\theta+1)(2,\theta^2+\theta+1)\ \ (3)=(2\theta+3)(4\theta^2-6\theta+25)\\ (5)=(5,\theta-2)(5,\theta^2+2\theta+8)\ \ (7)=(\theta+2)(7,\theta-1)^2\\ (11)=(11,\theta-5)(11,\theta^2+5\theta+29)\end{cases}$

PART I.—Continued.

		$d(\theta)$	Δ		h	Basis.	Units.	Factorization of Rational Primes.
145	$x^3-4x+9=0$	-1931	-1931	-1931	2	......	......	145 $\begin{cases}(2)=(2,\theta+1)(2,\theta^2+\theta+1)\ (3)=(-\theta^2-2\theta+2)(3,\theta)(3,\theta+1)\ (5)=(5)\\(7)=(7)\ (11)=(11)\end{cases}$
146	$x^3+5x+9=0$	-2687	-2687	-2687	2	......	$3\theta+4\pm k^2$	146 $\begin{cases}(2)=(2)\ (3)=(-\theta-1)(3,\theta)(3,\theta-1)\ (5)=(5,\theta-1)(5,\theta^2+\theta+1)\\(7)=(7)\ (11)=(11,\theta+3)(11,\theta^2-3\theta+3)\ (13)=(13)\end{cases}$
147	$x^3-5x+9=0$	$-7\cdot241$	$-7\cdot241$	-1687	1	......	$7\theta^2-20\theta+22$	147 $\begin{cases}2=2\ 3=(\theta+3)(\theta^2-3\theta+4)\ 5=(-\theta+1)(\theta^2+\theta-4)\\7=(-\theta+2)^2(\theta^2+\theta-5)\ 11=(-\theta-2)(\theta^2-2\theta-1)\end{cases}$
148	$x^3+6x+9=0$	$-3^3\cdot113$	$-3\cdot113$	-339	1	$\frac{\theta^2}{3},\theta,1$	......	148 : $2=(-\theta-1)(\theta^2-\theta+7)\ 3=\left(\frac{\theta^2+6}{3}\right)^2\left(\frac{\theta^2}{3}\right)\ 5=5$
149	$x^3-6x+9=0$	red.						149 :
150	$x^3+7x+9=0$	-3559	-3559	-3559	2	......	$\theta+1\pm k^2$	150 $\begin{cases}(2)=(2)\ (3)=(3,\theta)(3,\theta^2+1)\ (5)=(5)\ (7)=(7)\ (11)=(11)\\(13)=(\theta+2)(13,\theta+3)(13,\theta-5)\end{cases}$
151	$x^3-7x+9=0$	$-5\cdot163$	$-5\cdot163$	-815	1	......	.....	151 : $2=2\ 3=(-\theta+1)(-\theta+2)(-\theta-3)\ 5=(-\theta^2-\theta+7)^2(3\theta^2-9\theta+8)\ 7=7$
152	$x^3+8x+9=0$	red.						152 :
153	$x^3-8x+9=0$	-139	-139	-139	1	......	$\theta-2$	153 : $2=(-\theta+1)(\theta^2+\theta-7)\ 3=(-2\theta^2-2\theta+15)(2\theta^2-6\theta+5)$
154	$x^3+9x+9=0$	$-3^6\cdot7$	$-3^4\cdot7$	-567	1	$\frac{\theta^2}{3},\theta,1$	$\theta+1$	154 : $2=2\ 3=\left(\frac{\theta^2}{3}\right)^3(12\theta^2-11\theta+118)^*\ 5=\left(\frac{\theta^2+6}{3}\right)\left(\frac{11\theta^2-3\theta+3}{3}\right)$
155	$x^3-9x+9=0$	3^6	3^4	81	1	......	......	155 :
156	$x^3+9x+1=0$	$-3^3\cdot109$	$-3\cdot109$	-327	1	$\frac{\theta^2-\theta+1}{3},\theta,1$	$\theta,\ 9\theta+1$	156 : $2=2\ 3=\left(\frac{\theta^2-\theta+1}{3}\right)^2\left(\frac{7\theta^2-\theta+64}{3}\right)\ 5=\left(\frac{\theta^2+5\theta+1}{3}\right)\left(\frac{11\theta^2-2\theta+98}{3}\right)$
157	$x^3-9x+1=0$	$3^3\cdot107$	$3\cdot107$	321	1	$\frac{\theta^2-\theta+1}{3},\theta,1$	$\theta,\ \theta+3,\ \theta-3$	157 : $2=2\ 3=\left(\frac{\theta^2-\theta+1}{3}\right)^2(11\theta^2-\theta+100)$
158	$x^3+9x+2=0$	$-2^4\cdot3^3\cdot7$	$-2^2\cdot3^3\cdot7$	-756	1	$\frac{\theta^2+\theta}{2},\theta,1$	$\frac{5\theta^2-17\theta-4}{2}$	158 $\begin{cases}2=\left(\frac{-\theta^2-\theta}{2}\right)^2\left(\frac{59\theta^2-13\theta+534}{2}\right)\ 3=\left(\frac{\theta^2-\theta}{2}\right)^3\left(\frac{267\theta^2-59\theta+4834}{2}\right)^*\\5=5\ 7=(5\theta^2-\theta+45)^2(2\theta^2+14\theta+3)\end{cases}$
159	$x^3-9x+2=0$	$2^3\cdot3^3\cdot13$	$2^3\cdot3^3\cdot13$	2808	1	......	......	159 $\begin{cases}2=(-\theta-3)^2(7\theta^2-22\theta+6)\ 3=(-2\theta^2+6\theta-1)^3(-30\theta^2-22\theta+221)^*\\5=(\theta^2-3\theta+1)(-\theta^2+\theta+13)\ 7=7\ 11=11\end{cases}$
160	$x^3+9x+3=0$	$-3^5\cdot13$	$-3^5\cdot13$	-3159	1	......	......	160 $\begin{cases}2=2\ 3=(-\theta)^3(9\theta^2-3\theta+82)^*\ 5=5\ 7=(\theta+1)(\theta^2-\theta+10)\\11=11\ 13=(3\theta^2-\theta+28)(-2\theta^2+\theta+1)\end{cases}$
161	$x^3-9x+3=0$	$3^5\cdot11$	$3^5\cdot11$	2673	1	......	$3\theta-1$	161 $\begin{cases}2=2\ 3=(-\theta)^3(9\theta^2+3\theta-80)^*\ 5=(\theta-1)(\theta^2+\theta-8)\\7=(\theta-2)(\theta^2+2\theta-5)\ 11=(\theta^2+2\theta-4)^2(4\theta^2-13\theta+5)\end{cases}$
162	$x^3+9x+4=0$	$-2^2\cdot3^3\cdot31$	$-2^2\cdot3^3\cdot31$	-3348	1	......	......	162 $\begin{cases}2=(16\theta^2-7\theta+147)^2(10\theta^2-37\theta-18)\\3=(3\theta^2-\theta-1)^3(6244913\theta^2-2718291\theta+57387437)^*\\5=(2\theta+1)(4\theta^2-2\theta+37)\ 7=(-2\theta^2+6\theta+3)(78\theta^2-34\theta+717)\\11=(1916\theta^2-834\theta+17607)(48\theta^2-14\theta-3)\\13=(12477\theta^2-5431\theta+114657)(133\theta^2+51\theta-3)\end{cases}$
163	$x^3-9x+4=0$	$2^2\cdot3^3\cdot23$	$3^3\cdot23$	621	1	$\frac{\theta^2+\theta}{2},\theta,1$	$-4\theta^2+26\theta-3$	163 : $2=\left(\frac{\theta^2+\theta-8}{2}\right)(\theta-1)\ 3=(2\theta-1)^3(488\theta^2+222\theta-4291)^*\ 5=5$

PART I.—Continued.

		$d(\theta)$	Δ		h	Basis.	Units.	Factorization of Rational Primes.
164	$x^3+9x+5=0$	$-3^3.7.19$	$-3^3.7.19$	-3591	1	.....	$-4\theta^2+22\theta+13$	164 $\begin{cases} 2 = 2\ \ 3=(-2\theta-1)^3(52\theta^2-28\theta+483)^*\ \ 5=(-\theta)(\theta+1)(2\theta^2-\theta+19) \\ 7=(2\theta^2-\theta+18)^2(-\theta^2+5\theta+3)\ \ 11=(11\theta^2-6\theta+102)(3\theta^2+7\theta+3) \\ 13=13 \end{cases}$
165	$x^3-9x+5=0$	$3^3.83$	$3^3.83$	1241	1	......	$-\theta^2+4\theta-2$	165 : $2=2\ \ 3=(\theta-1)^3(5\theta^2+3\theta-43)^*\ \ 5=(-\theta)(-\theta+3)(-\theta-3)\ \ 7=7$
166	$x^3+9x+6=0$	$-2^4.3^5$	-3^5	-243	1	$\equiv 11$	......	166 :
167	$x^3-9x+6=0$	$2^3.3^5$	$2^3.3^5$	1944	1	......	......	167 $\begin{cases} 2=(\theta-1)^2(-3\theta^2-2\theta+26)\ \ 3=(\theta^2-5\theta+3)^3(14007\theta^2+9885\theta-119087)^* \\ 5=5\ \ 7=(-\theta^2-\theta+7)(-\theta^2+\theta+1) \end{cases}$
168	$x^3+9x+7=0$	$-3^3.157$	$-3^3.157$	-4239	1	......	$\theta^2-2\theta-2$	168 $\begin{cases} 2=2\ \ 3=(\theta+1)^3(15\theta^2-11\theta+143)^*\ \ 5=5\ \ 7=(-\theta)(\theta^2+9) \\ 11=(\theta^2-\theta+9)(\theta^2+2\theta+2)\ \ 13=13\ \ 17=(-\theta+1)(\theta^2+\theta+10) \end{cases}$
169	$x^3-9x+7=0$	$3^3.59$	$3^3.59$	1593	1	......	$\theta-1$	169 $\begin{cases} 2=2\ \ 3=(\theta-2)^3(3\theta^2+7\theta-10)^*\ \ 5=(-\theta^2-3\theta+4)(-\theta^2+\theta+3) \\ 7=(-\theta)(-\theta+3)(-\theta-3) \end{cases}$
170	$x^3+9x+8=0$	$-2^2.3^3.43$	$-2^2.3.43$	-516	1	$\frac{\theta^2+\theta+1}{3}, \theta, 1$	......	170 : $2=(\theta+1)^2(6\theta^2-5\theta+58)\ \ 3=\left(\frac{\theta^2+\theta+1}{3}\right)^2\left(\frac{10\theta^2-8\theta+97}{3}\right)\ \ 5=5$
171	$x^3-9x+8=0$	red.						171 :
172	$x^3+10=0$	$-2^2.3^3.5^2$	$-2^2.3.5^2$	-300	1	......	$-3\theta^2-6\theta+1$	172 : $2=(-\theta-2)^3(-3\theta^2-6\theta+1)^*\ \ 3=\left(\frac{\theta^2-\theta+1}{3}\right)^2\left(\frac{\theta^2+2\theta+1}{3}\right)$

PART II.

		$d(\theta)$	Δ		h	Basis.	Units.	Factorization of Rational Primes.
1	$x^3-x^2+x+1=0$	$-2^2 . 11$	$-2^2 . 11$	-44	1	......	......	1 :
2	$x^3+x^2+x+2=0$	-83	-83	-83	1	......	$\theta+1$	$2:2=(-\theta)(\theta^2+\theta+1)$
3	$x^3-x^2+x+2=0$	-139	-139	-139	1	......	$\theta+1$	$3:2=(-\theta)(\theta^2-\theta+1)$ $3=(-\theta+1)(\theta^2+1)$
4	$x^3-x^2-x+2=0$	-59	-59	-59	1	......	$\theta-1, \theta+1$	$4:2=(-\theta)(\theta^2-\theta-1)$
5	$x^3+x^2+2x+1=0$	-23	-23	-23	1	......	......	5 :
6	$x^3-x^2+2x+1=0$	$-3 . 29$	$-3 . 29$	-87	1	......	θ	$6:2=2$
7	$x^3+x^2-2x+1=0$	-31	-31	-31	1	......	......	7 :
8	$x^3-x^2-2x+1=0$	7^2	7^2	49	1	......	......	8 :
9	$x^3-x^2+2x+2=0$	$-2^3 . 5^2$	$-2^3 . 5^2$	-200	1	......	......	$9:2=(-\theta)^2(\theta^2-2\theta+3)$ $3=3$
10	$x^3+x^2-2x+2=0$	$-2^3 . 19$	$-2^3 . 19$	-152	1	......	$\theta^2-\theta+1$	$10:2=(-\theta)^2(-\theta^2-2\theta+1)$ $3=3$
11	$x^3-2x^2+x+2=0$	$-2^2 . 29$	$-2^2 . 29$	-116	1	......	......	$11:2=(\theta+1)^2(-\theta)$ $3=3$
12	$2x^3-x^2+x+2=0$ $x^3-x^2+2x+8=0$	-503 $-2^2 . 503$	-503	-503	1	$\dfrac{\theta^2+\theta}{2}, \theta, 1$	......	$12:2=(2\theta^2-5\theta+11)(\theta^2-2)$ $3=3$ $5=(2\theta+3)(4\theta^2-10\theta+23)$
13	$x^3-2x^2+2x+2=0$	$-2^2 . 51$	$-2^2 . 51$	-204	1	......	$-\theta^2-5\theta+7$	$13:2=(-\theta)^3(-\theta^2+5\theta-7)$* $3=(-\theta+1)(\theta+1)(\theta^2-2\theta+3)$
14	$x^3+2x^2-2x+2=0$	$-2^2 . 67$	$-2^2 . 67$	-268	1	......	$\theta+3$	$14:2=(-\theta)^3(\theta+3)$* $3=(-\theta+1)(\theta^2+3\theta+1)$
15	$x^3-2x^2-2x+2=0$	$2^2 . 37$	$2^2 . 37$	148	1	......	$\theta+1, \theta-1$	$15:2=(-\theta)^3(2\theta^2-3\theta-5)$*
16	$2x^3+x^2+2x+2=0$ $x^3+x^2+4x+8=0$	$-2^2 . 89$ $-2^4 . 89$	$-2^2 . 89$	-356	1	$\dfrac{\theta^2+\theta}{2}, \theta, 1$	.....	$16:2=\left(\dfrac{\theta^2-\theta+4}{2}\right)^2\left(\dfrac{-\theta^2-3\theta-2}{2}\right)$ $3=3$ $5=5$
17	$2x^3-x^2+2x+2=0$ $x^3-x^2+4x+8=0$	$-2^2 . 157$ $-2^4 . 157$	$-2^2 . 157$	-628	1	$\dfrac{\theta^2+\theta}{2}, \theta, 1$	......	$17\begin{cases} 2=\left(\dfrac{\theta^2+\theta}{2}\right)^2\left(\dfrac{25\theta^2-55\theta+166}{2}\right) \; 3=(5\theta^2-11\theta+33)(\theta^2-3\theta-5) \\ 5=\left(\dfrac{\theta^2-\theta+2}{2}\right)\left(\dfrac{3\theta^2-7\theta+18}{2}\right) \; 7=7 \end{cases}$
18	$2x^3+x^2-2x+2=0$ $x^3+x^2-4x+8=0$	$-2^2 . 3 . 43$ $-2^4 . 3 . 43$	$-2^2 . 3 . 43$	-516	1	$\dfrac{\theta^2+\theta}{2}, \theta, 1$	......	$18\begin{cases} 2=\left(\dfrac{\theta^2+3\theta}{2}\right)^2\left(\dfrac{7\theta^2-15\theta+18}{2}\right) \; 3=(9\theta^2-19\theta+23)^2(-6\theta^2+26\theta+139) \\ 5=5 \end{cases}$
19	$2x^3-x^2-2x+2=0$ $x^3-x^2-4x+8=0$	$-2^2 . 53$ $-2^4 . 53$	$-2^2 . 53$	-212	1	$\dfrac{\theta^2+\theta}{2}, \theta, 1$	$\theta^2+\theta-3$	$19:2=\left(\dfrac{-\theta^2-\theta+4}{2}\right)^2\left(\dfrac{\theta^2-3\theta+2}{2}\right)$ $3=3$